Announcements

1) Exam Thursday after Thanksgiving

Recall: we were solving

$$r^2 f''(r) + r f'(r) + \alpha r^2 f(r) = 0$$

We supposed

$$f(r) = \sum_{n=0}^{\infty} a_n r^n$$

and obtained

$$a_1 r + \sum_{n=2}^{\infty} \left(n^2 a_n - \alpha a_{n-2}\right) r^n = 0$$

Since a "polynomial" is zero only if all coefficients are zero, we get

$$\boxed{\begin{aligned} &a_1 = 0, \\ &n^2 a_n - \alpha a_{n-2} = 0, \; n \geq 2. \end{aligned}}$$

$a_1 = 0,$

$9a_3 - \alpha a_1 = 0$

(second equation, $n=3$)

so $9a_3 = 0$, $a_3 = 0$.

(2^{nd} equation, $n=5$)

$25a_5 - \alpha a_3 = 0$, but

$a_3 = 0$, so

$25a_5 = 0$ and $a_5 = 0$.

Using recursion,

$$\boxed{a_n = 0 \text{ for } n \text{ odd}}$$

For the remainder of the coefficients, $n=2k$ for $k=1,2,3,\ldots$ (n even)

and we can choose any value for a_0 since it does not appear in the sum other than through recursion.

$$\boxed{(2k)^2 a_{2k} - \alpha a_{2(k-1)} = 0}$$

for $k=1,2,3,\ldots$

$k=1$

$$4a_2 - \alpha a_0 = 0$$

so $a_2 = \dfrac{\alpha a_0}{4}$

$k=2$

$$16a_4 - \alpha a_2 = 0$$

so $a_4 = \dfrac{\alpha a_2}{16}$

$$= \frac{\alpha}{16}\left(\frac{\alpha a_0}{4}\right)$$

$$= \frac{\alpha^2}{4\cdot 4\cdot 4} a_0$$

$k = 3$

$$36a_6 - \alpha a_4 = 0$$

$$a_6 = \frac{\alpha a_4}{36}$$

$$= \frac{\alpha}{36}\left(\frac{\alpha^2 a_0}{64}\right)$$

$$= \frac{\alpha^3 a_0}{9 \cdot 4 \cdot 4 \cdot 4 \cdot 4}$$

$k = 4$

$$64 a_8 - \alpha a_6 = 0$$

$$a_8 = \frac{\alpha a_6}{64}$$

$$= \frac{\alpha}{64} \cdot \frac{a_0 \, \alpha^3}{(9 \cdot 4) \, 4^3}$$

$$= \frac{\alpha^4 \cdot a_0}{(16 \cdot 9 \cdot 4) \, 4^4}$$

$$\boxed{a_{2k} = \frac{\alpha^k a_0}{(k!)^2 \, 4^k}}$$